





Saint Petersburg State University, Volkhovskiy Pereulok, 3, lecture hall 403, Saint Petersburg, Russia

### SECOND INTERNATIONAL SCHOOL FOR STUDENTS AND YOUNG SCIENTISTS

# **2016** Non-Equilibrium & High-Temperature Gas Flows



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#### The school is organized by

- Saint Petersburg State University
- Khristianovich Institute of Theoretical and Applied Mechanics
- Russian Science Foundation

#### **Organizing Committee**

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#### **Final Program**

The School takes place at the Saint Petersburg State University, Saint Petersburg, Russia.

Registration, Coffee Breaks: Volkhovskiy Pereulok, 3, room 309

Lectures: Volkhovskiy Pereulok, 3, room 305

#### **Detailed Program**

Monday, September 26, 2016

9:00 – 16:00 Registration, School opening, Lectures

#### Tuesday, September 27, 2016

10:00 - 17:00 Lectures

#### Wednesday, September 26, 2016

10:00 - 16:00 Lectures

Monday, September 26, 2016			
Registration		9.00 - 10.00	
School opening		10.00 - 10.20	
Cł			
Sergei Utyuzhnikov	Introduction to High-temperature Thermodynamics and Its Applications to High-speed Flows	10.20 – 11.10	
Coffee break		11.10 - 11.30	
Vladimir Titarev	Parallel numerical methods for solving model kinetic equations in three space dimensions	11.30 – 12.20	
Yurii Shevelev	Numerical modeling of 3-D Hypersonic Aerodynamics and Heat Transfer Problems	12.20 – 13.10	
Lunch		13.10 - 14.30	
Ch			
Andrea Lombardi	CO2 dynamics in gaseous mixtures under nonequilibrium conditions	14.30 – 15.20	
Gennady Markelov	High-altitude aerothermodynamics of (re-)entry vehicles and spacecraft	15.20 – 16.10	

Tuesday, September 27, 2016			
Ch			
Svetlana Starikovskaia	Nanosecond discharges at high volumetric energy release: hydrodynamic and kinetic effects	10.00 – 10.50	
Valentin Bityurin	Comparative analysis of heat release and dynamics effects in magnetoplasma aerodynamics	10.50 – 11.40	
Coffee break		11.40 - 12.00	
lgor Mashek	Microwave discharge guided by laser spark and its applications for supersonic aerodynamics	12.00 – 12.50	
	12.50 - 14.10		
Irina Krassovskaya	Reflection and diffraction of shock - wave configurations	14.00 – 14.50	
Oleg Shatalov	Shock waves in gases. Contemporary kinetic experiment	14.50 – 15.40	
Alexander Uvarov	Propagation of small hydrodynamic perturbations in subsonic nonuniform nonequilibrium flows and stability analysis	15.40 – 16.30	

Wednesday, September 28, 2016			
Ch			
Aldo Frezzotti	Modelling non-equilibrium flows in two-phase systems	10.00 – 10.50	
Albert Stasenko	Multiphase flow physics	10.50 – 11.40	
Coffee break		11.40 - 12.00	
Maria Rydalevskaya	Statistical Description of Gases with Physical and Chemical Processes and Determination of Equilibrium Plasma Composition	12.00 – 12.50	
	12.50 - 14.10		
Gilberto Medeiros Kremer	The Boltzmann equation as a tool for the analysis of gravitational problems	14.00 – 14.50	
Valerii Shematovich	Dissipation of the planetary atmospheres in the Solar and extrasolar planetary systems	14.50 – 15.40	

Presentation requirements

English and Russian are official languages of the School.

The Conference hall is equipped with a laptop, running Windows 7 with MS Office 2010 (\*.ppt and \*.pptx) and Adobe Reader 11 (\*.pdf), connected to a data projector. The Organizing Committee cannot guarantee correct work of the projector with attendees' laptops, so using the School laptop is highly encouraged.

Invited talks will be 50 minutes long, including a 5 minute question-answer session.

#### School venue

Lectures will take place in the Graduate School of Management, Saint Petersburg State University, Saint Petersburg, Russia. The address of the Graduate School of Management (GSOM SPbU) is Volkhovskiy Pereulok, 3 (see the map below). Registration of attendees will begin at 9:00 on September 26. Please do not forget to wear your name badge which will be given to you on the registration.

Open access WiFi network will be available at the School venue (connection name: spbu.edu).



- 1. Graduate School of Management
- 2. Sokos Vasilievsky Hotel
- 3. SPbVergaz Hotel
- 4. Marketplace cafe
- 5. Students canteen cafe
- 6. Dve palochki cafe
- 7. Facultet cafe

#### Abstracts of invited talks

#### Introduction to High-temperature Thermodynamics and Its Applications to High-speed Flows

#### S.V. Utyuzhnikov

University of Manchester, Moscow Institute of Physics & Technology email: S.Utyuzhnikov@manchester.ac.uk

Basic foundations of high-temperature thermodynamics are addressed. The talk starts from the thermodynamics of gas mixtures, description of equilibrium and nonequilibrium chemical reactions in high-speed flows. Elements of statistical thermodynamics are introduced to describe the behavior of molecules at high temperatures with the use of Maxwell-Boltzmann distribution. The effects of multiple temperature and multicomponent diffusion are analyzed in application to re-entry space vehicles.

#### Numerical modeling of 3-D Hypersonic Aerodynamics and Heat Transfer Problems

#### Yu. D. Shevelev

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The perspectives in the development of the new launch vehicles are considered. Hypersonic flows over real space configurations represent a substantial problem from the point of view of the development of new and more effective mathematical models, numerical algorithms and the use of modern computer systems. For numerical simulation of external flow fields past real form bodies it is necessary to construct the geometry, to design a discrete set-grid, to provide the mathematical model of the initial value problem, to approximate the governing equation by numerical ones, to design a computational algorithm, to realize the flow field, to establish a feed-back of obtained results with experiment, analytical and benchmark problems and so on.

The traditional techniques of geometry generation are the spline interpolation and algebraic methods and others. For fluid dynamics problems it is useful to represent the surface by algebraic methods. The main ideas of the abovementioned methods are implemented in the ACAD system (design system for aerodynamic purposes). One of the basic problem is the grid generation that takes into account geometrical and physical features of the flow field. The physical region is divided into sub-regions and within each sub-region a structured grid is generated. The methods of 3-D generation are based on the procedure of elliptical and parabolic generators.

Simulation of 3-D gas flow is carried out within the framework of different mathematical models: Euler and Navier-Stokes equations and also on the basis of a "thin" layer approach. The algorithms are capable of capturing a variety of complex

flow features such as shock waves and contact discontinuities, vortex shedding, shock and boundary-layer interactions, flow separation.

The thermo-chemical models of the processes occurring in the shock layer includes the different types of processes: chemical reactions, dissociation, ionisation and recombination, exchange reactions, processes of vibration energy-exchange between various levels of molecules, influence of the vibration relaxation on the chemical reactions (CV-processes), processes of excitation and deactivation of the electronic states of molecules, radiation processes with involvement of excited particles. For a correct prediction of the heat transfer and the surface temperatures for a space vehicle during an entry careful examination of the experimental catalytic properties of the thermal protection covering is needed. The algorithms are realized for gas-phase and gas wall interaction models of various complexities from perfect gas to multi-temperature, multi-species and thermo-chemically non-equilibrium gas medium.

For unsteady 3-D N-S equations the solvers (high accuracy, compact monotone schemes, without the "artificial" viscosity, with dispersion control) are used on adaptive grids.

Finally some of the results of the CFD applications are presented. Solutions of hypersonic flow problems past "Soyuz", "Mars", "Buran" and geometries of reusable launch vehicle (RVL) are displayed. The results aim to show the possibilities of mathematical models and numerical methods of flow fields modeling for space vehicles. The algorithms are implemented both in sequential computers and parallel ones.

#### High-altitude aerothermodynamics of (re-)entry vehicles and spacecraft

#### **Gennady Markelov**

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Knowledge of high-altitude aerothermodynamics is important for a number of tasks, for example, performing an entry into a planetary atmosphere, aerocapture and aerobraking maneuvers, and modeling of space debris demise in Earth atmosphere. The latter draws attention due to a growing number of space debris and as a result a spacecraft design-for-demise strategy is studied actively. A new challenge is emerging inflatable and deployable aerodynamic decelerators that provide mass advantages for future space missions. Numerical modelling and engineering assessment are discussed for high-altitude aerothermodynamics of capsules and spacecraft with a complex shape. Conventional capsule and deployable aerodynamic decelerators.

#### CO2 dynamics in gaseous mixtures under nonequilibrium conditions

#### Andrea Lombardi

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The development of kinetic models of gaseous systems involving CO2, where the energy transfer is modelled explicitly at a state-to-state level of detail, is a necessary advance, that requires much insight into the dynamics of the inelastic collisions and the prompt availability of state-specific energy transfer probabilities and rate coefficients. These have to be calculated directly by simulation of molecular collisions, covering the wide range of conditions (energy, temperature) typical of gas flows. Here, a simulation approach to the study of vibrational state-specific collision cross sections and rate coefficients of gas phase processes will be illustrated with reference to the CO2+CO2 and CO2+N2 systems [1-3].

#### REFFERENCES

 [1] M. Bartolomei, F. Pirani, A. Lagana, A. Lombardi, A full dimensional grid empowered simulation of the CO2 + CO2 processes, J. Comput. Chem. 2012, 33: 1806

[2] A. Lombardi, N. Faginas-Lago, L. Pacifici, A. Costantini, Modeling of energy transfer from vibrationally excited CO2 molecules: cross sections and probabilities for kinetic modeling of atmospheres, flows, and plasmas, J. Phys. Chem. A 2013, 117: 11430

[3] A. Lombardi, F. Pirani, A. Laganà, M. Bartolomei, Energy Transfer Dynamics and Kinetics of Elementary Processes (Promoted) by Gas-Phase CO2-N2 Collisions: Selectivity Control by the Anisotropy of the Interaction, J. Comput. Chem. In press.

### Parallel numerical methods for solving model kinetic equations in three space dimensions

#### Vladimir Titarev

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In recent years there has been rapid development of numerical methods and associated computer codes for solving the Boltzmann kinetic equation with the exact or model collision integrals in three space dimensions. In this lecture we will briefly describe methods for solving the kinetic equation with the Shakhov model collision integral. This equation is a six-dimensional differential-integral equation and is thus difficult to solve numerically. We will consider three main topics. The first one is the high-order non-oscillatory approximation of the convective operator and conservative procedure for the calculation of the model collision integral. The second topic is construction of an efficient implicit time evolution method for steady-state problems. Finally, a two-level MPI+OpenMP parallel implementation on modern parallel computers will be outlined.

The practical implementation of the discussed topics will be illustrated using the recent "Nesvetay-3D", developed by the author. Examples of both steady-state high-speed flow calculations as well as time-dependent flows will be provided.

### Nanosecond discharges at high volumetric energy release: hydrodynamic and kinetic effects

#### Svetlana Starikovskaia

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Physics and chemistry of high-voltage nanosecond discharges at high volumetric energy release will be discussed. A review of experimental techniques (current, voltage, measurements, sub-microsecond shadography, energy emission spectroscopy, LIF/TALIF) will be given together with detailed analysis of kinetics of the discharges. In nanosecond gas discharges, the energy goes into internal degrees of freedom via molecules and atoms colliding with electrons at a time scale much less than the characteristic time of chemical reactions within the system. High densities of electronically excited species provide exceptionally high rate of ET relaxation, leading to local increase of translational gas temperature in zones of high energy release over tens of nanoseconds while leaving the rest of the gas or surface non-heated on the same time scale. This provides an exceptional breakthrough for application of nonequilibrium plasma at elevated pressures, allowing triggering biological processes or chemistry locally, managing relative influence of radical chemistry and heat release by (i) electric field in the discharge; (ii) heat release/removal in the afterglow on the time scale shorter or comparable with a characteristic time of chemical reactions.

### Comparative analysis of heat release and dynamics effects in magnetoplasma aerodynamics

#### V.A. Bityurin

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The problem of the comparative estimation of the heat release and dynamics effects in gas discharge plasma affected with external electrical and magnetic fields is considered. This problem arises during the efficiency evaluation of the plasma aerodynamics technique proposed for the flow control purposes. The main attention is paid in this study to the mechanism of remote flow control by the momentum release in the gasdynamics flow field using controlled external electrical and magnetic fields created by on-board system. The action of the Lorentz force  $F=\sum qk \cdot (E + wk \times B)$  is undouble followed by Joule dissipation of some portion of the work done by this force. Nevertheless, one can see that in many publications on the analysis of plasma actuators the heat release effect is neglected in compare with 'direct' dynamic effect of electrodynamics body forces. In this paper we present rather robust estimation of comparable role of the heat and dynamic effects and couple of examples of the numerical simulation of typical magnetoplasma aerodynamics situation which demonstrate the importance both the heat and dynamics impacts of electrodynamics body force.

### Microwave discharge guided by laser spark and its applications for supersonic aerodynamics

#### I.Ch. Mashek

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The methods of realizations, investigations and possible applications for selfsustaining MW discharges and Laser-guided MW discharges in supersonic gas flows will be presented. The last experiments performed at JIHT (Moscow) and SPBSU (Saint-Petersburg) with MW-Laser energy deposition in air flow with M=2,1-3,0 and static pressure 50-150 mbar will be presented and discussed.

#### **Reflection and diffraction of shock - wave configurations**

#### M. K. Berezkina, I. V. Krassovskaya

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There are two phenomena, which can result from interaction of a shock wave (SW) with a ramp. The SW reflection phenomenon takes place in case of the angle of incidence less than 90°. If the angle of incidence is more than 90°, a SW diffracts over a ramp. It is important to note, that diffraction phenomena don't exist in steady flows. Besides, the term "collision of two SW" loses any meaning for steady flow. This reveals the principal difference between the processes of interaction of two gasdynamic discontinuities in steady and pseudo-steady flows. In the steady case we deal with the shock-wave configurations forming in the internal or external flows regardless of the condition of their origin. The pseudo-steady and unsteady shock-wave interactions have always the point corresponding to the beginning of the process. Therefore, we would like to separate the range of problems arising in steady flows.

The key to understanding processes of reflection and diffraction of a single SW in

pseudo-steady flow lies in the idea that the whole process should be considered as a combination of two subprocesses: an incident SW front reflection or diffraction and an incident shock induced flow deflection around the leading edge of a ramp. As a result of interference, in the case of SW diffraction a complex inner structure containing a separation zone, a vortex and a shock-wave configuration of stagnation arises (Skews 1967, Hillier 1995). In the case of SW reflection, the variety of the reflection types is formed. On the base of the idea of interference the classification of the types of SW reflection in pseudo-steady flow was suggested in Semenov et al. 2009.

In the present paper some results of the studies of shock wave reflection and diffraction in unsteady flows are reported. Considering processes which take place on the second surface of a double wedge, one should take into account that, not an incident SW but a shock-wave configuration will reflect or diffract over this surface. Thus, the incident wave can no longer be considered as a single wave without related discontinuities.

Diffraction of both two-shock and three-shock configuration is considered. Fig.1 shows the flow field at the interaction of a SW with the double-faced convex wedge. The three-shock Mach reflection configuration has been formed on the first reflecting surface. Taking into account the principal difference of the flow pattern for diffraction of two-shock and three-shock configuration, it is possible to recognize what kind of reflection (regular or Mach) occurred on the first surface of a double wedge. It can be helpful if the reflection

conditions are near critical values for regular reflection.



The results of investigation of a SW reflection from concave and convex double wedge are given and the sequence of the reflection types is considered. In Fig.2 the

data of the SW reflection over the concave double wedge is presented. The Mach reflection configuration forms on the first wedge. It interacts with the second wedge and as a result, the double Mach reflection configuration occurs. Note that, under the same condition the reflection of the SW from a single wedge is regular. It is shown that, in unsteady flows one should not expect that transition between regular and Mach reflection will proceed at the parameters that are

found for pseudo-steady flows.

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#### Shock waves in gases. Contemporary kinetic experiment

#### **O.P. Shatalov**

#### Institute of Mechanics of Lomonosov Moscow State University, Russia email: shatalov@imec.msu.ru

Under laboratory conditions, shock tubes are most often used to study the shock waves. A strong shock wave in the tube can heat the gas to several tens thousands degrees. Today this is the most available way to obtain high temperatures. The shock tube is a relatively simple laboratory device, but together with modern recording equipment, it allows us to research on a wide shock wave velocity range, in different spectra of emission and absorption of atoms and molecules, to monitor the populations of the vibrational and electronic levels of atoms and molecules, to measure the electron concentration in the gas, etc. Two examples of such studies presented in this paper are the investigation dissociation kinetics of oxygen molecules under thermally nonequilibrium conditions [1] and the analysis of argon emission at Mach numbers of the order of 20 [2].

The oxygen absorbance was studied at wavelengths 200–270 nm in Schumann-Runge system behind the front of a strong shock wave. Using these data, the vibrational temperature Tv behind the front of shock waves was measured at temperatures 4000–10 800 K in undiluted oxygen. Determination of Tv was based on the measurements of time histories of absorbance for two wavelengths behind the shock front and on the results of detail calculations of oxygen absorption spectrum. Solving the system of standard quasi-one-dimensional gas dynamics equations and using the measured vibrational temperature, the time evolution of oxygen concentration and other gas parameters in each experiment were calculated. Based on these data, the oxygen dissociation rate constants were obtained for thermal equilibrium and thermal non-equilibrium conditions. Furthermore, the oxygen vibrational relaxation time was also determined at high temperatures. Using the experimental data, various theoretical and empirical models of high-temperature dissociation were tested, including the empirical model proposed in the present work.

Experimental study of argon radiation behind the shock front is presented at velocities up to 7 km/s and the pressure ahead of the front 0.25 and 1.0 Torr. The integral radiation of argon on the range from 200 to 900 nm, as well as the temporal evolution of individual spectral lines, characterizing the population of argon emitting excited levels, were both measured. These data allow us to estimate the temperature dependence of processes that control radiation: the production of primary electrons in Ar-Ar collisions, the rate of exponential growth of radiation in collisions Ar-e, and the speed of depopulation of the emitting level. Proposed algorithm allows us to describe the experimentally observed temporal characteristics of the Ar emission lines. The algorithm uses the 4-level model of Ar. The calculations assume the transition from the ground to excited state of argon to be a result of Ar-Ar and Ar-e collisions, while the deactivation of the excited state occurs as a result of collisions and spontaneous emission. A simplified set of reactions is used in the simulation of argon excitation and ionization.

#### REFFERENCES

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[2] Быкова Н. Г., Забелинский И. Е., Козлов П. В., Левашов В. Ю., Ибрагимова Л. Б., Шаталов О. П. Исследование радиационных характеристик аргона за фронтом сильной ударной волны //Физико-химическая кинетика в газовой динамике. 2015. Т.16, вып. З. <u>http://chemphys.edu.ru/issues/2015-16-</u> <u>3/articles/577/</u>

### Propagation of small hydrodynamic perturbations in subsonic nonuniform nonequilibrium flows and stability analysis

#### A.V. Uvarov

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Propagation of small perturbations in nonuniform and nonequilibrium flows has two main features. First, nonuniformity of the medium results in more complicated set of independent hydrodynamic modes, defined by linear system of differential equation with variable coefficients. Second, perturbations (sound and thermal modes) are amplified in nonequilibrium medium. However, the eigenfunctions for arbitrary perturbation can be calculated numerically using the linear approximation. Feedback in the flow can be caused by reflection from the boundaries or from unperturbed profile of hydrodynamic parameters. Standard problem of stability of detonation and combustion waves in 1D approximation is a simple example of such analysis. An alternative approach to stability problem is to perform perturbation calculations using standard software packages. However, this approach sometimes leads to erroneous results due to specific boundary conditions for unperturbed flow, which can significantly affect propagation of small perturbations.

#### Modelling non-equilibrium flows in two-phase systems

#### Aldo Frezzotti

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Two-phase flows in which a gas/vapor is in contact with a liquid phase are of great practical and theoretical interest. Their mathematical and numerical treatment is far from being trivial because, although the bulk regions of the liquid and vapor phases are well described by hydrodynamic equations, the presence of sharp interfaces produces significant deviations from hydrodynamic behavior. The interface region can be divided into two well defined parts. The liquid-vapor interface itself, where the density suffers changes of orders of magnitude in the space of a few molecular diameters and a kinetic layer (Knudsen layer) to be described by kinetic equations. Such kinetic region is always present in non-equilibrium conditions. However, its importance and structure depends on the flow regime. For instance, quite thick Knudsen layers are formed on top of the condensed phase during laser induced ablation or intense evaporation, in general. Although the kinetic theory community has concentrated on studying the stationary, spatially one-dimensional Knudsen layer structure and, to a lesser extent, the kinetic boundary conditions models to adopt at the liquid-vapor interface, there are problems where the kinetic equation governing the vapor phase has to be coupled with the liquid phase along a moving and shape changing interface. The talk will summarize the well established results about the Knudsen layer structure in evaporation and condensation flows for simple mono/polyatomic gases and mixtures and will discuss models which have the capability of a unified description of both phases (Diffuse Interface Models, Enskog-Vlasov kinetic equation), highlighting their properties through specific examples and test problems.

#### **Multiphase flow physics**

#### A.L. Stasenko

#### Central Aerohydrodynamic Institute named after N.E. Zhukovsky, Moscow, Russia email: stasenko@serpantin.ru

A principal feature of polydisperse flow is the mass and thermal inertia of particles, accelerated by a carrying medium, gas or liquid. The irreversible interaction of phases is accompanied with the variety of physical processes, namely particle/medium exchange with mass, momentum and energy which result in the sound waves dispersion and absorption, antiparticle collisions, particulates impingement on the solid bodies flown around. Small dimensions of a disperse material cause specific optical characteristics of the mixture and demand to consider rarefaction of the carrying gas. Also, the coupling of these effects is essential, e.g. the mutual influence of the angular and straightforward movement of a particle. These physical phenomena are of principal interest for a flyer in dusted atmospheres of planets, in the supercooled water-crystals or in high altitude volcanic eruption clouds, and in numerous technologies. The aims of the lecture are physic-mathematical simulation of the mentioned processes and some analytical and numerical results obtained on a basis of these models.

#### Statistical Description of Gases with Physical and Chemical Processes and Determination of Equilibrium Plasma Composition

#### Maria A. Rydalevskaya

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Multicomponent gas systems, resulting from thermal ionization of spatially uniform mixtures, are assumed to be so rarefied that even after their multiple ionization they can be regarded as ideal gases. The method proposed for the equilibrium composition determination of these mixtures makes use of the fundamental principles of statistical physics. Equilibrium concentrations of the mixtures components are determined as a result of integration over the spaces of impulses and summation over the electronic energy levels of the distribution functions. These functions correspond to entropy maximum. To determine unknown parameters the systems of the equations, corresponding to the normalization conditions, are deduced. It is shown that the systems may be reduced to one algebraic equation if

equilibrium temperature is known. Numeral method to solve this equation is proposed.

#### The Boltzmann equation as a tool for the analysis of gravitational problems

#### **Gilberto M. Kremer**

Departamento de Física, Universidade Federal do Paraná Curitiba, Brazil email: kremer@fisica.ufpr.br

One of the problems analyzed here refers to Jeans instability. The search for structure formation from gas clouds is an old subject that goes back to 1902 when Jeans [1] used the system of phenomenological equations of mass and momentum densities together with the Poisson equation and showed that small perturbations in the mass density, pressure, velocity and gravitational potential in a static background with wavenumber smaller than the Jeans wavenumber could evolve with time. In terms of balance of forces the fluctuations can grow in time if the inwards directed gravitational force is larger than the outwards directed internal pressure of the gas. Jeans theory describes the gravitational instability of selfgravitating systems by searching for conditions that small perturbations can grow and leads to a collapse of the system. It was formulated before the knowledge of the Universe expansion and one has to take into account the Jeans "swindle", which imposes that the Poisson equation is valid only for the perturbations, since the background solution of constant mass density, pressure, gravitational potential and vanishing velocity satisfy the balance equations of mass and momentum densities, but not the Poisson equation. Here the dynamics of self-gravitating fluids is analyzed within the framework of a collisionless Boltzmann equation [2,3,4,5] in the presence of gravitational fields and Poisson equation. The equilibrium distribution function takes into account the expansion of the Universe and a pressureless fluid in the matter dominated Universe. Without invoking Jeans "swindle" a dispersion relation is obtained by considering small perturbations of the equilibrium values of the distribution function and gravitational potential. The collapse criterion, which happens in an unstable region where the solution grows exponentially with time, is determined from the dispersion relation.

The other problem concerns the analysis of self-gravitating systems of ideal gases in the post-Newtonian approximation [6]. We obtain the equilibrium relativistic distribution function (the so-called Maxwell-Jüttner distribution function) at first order in the post-Newtonian approximation within the framework of general relativity. Taking into account the aforesaid distribution function, we compute the particle four-flow and energy-momentum tensor. We focus on the search of static solutions for the gravitational potentials with spherical symmetry. In doing so, we obtain the density, pressure and gravitational potential energy profiles in terms of dimensionless radial coordinate by solving the aforesaid equations numerically. Due its physical relevance, we also find the galaxy rotation curves using the post-Newtonian approximation. We join two different kinds of static solutions in order to account for the linear regime near the center and the typical flatten behavior at large radii as well.

#### REFFERENCES

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[2] G. M. Kremer, An introduction to the Boltzmann equation and transport processes in gases, Springer, Berlin, 2010

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## Dissipation of the planetary atmospheres in the Solar and extrasolar planetary systems

#### V.I. Shematovich

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The escape (or dissipation) from a planetary atmosphere of molecules with thermal energies greater than the gravitational binding energy is referred to as thermal escape. Since the prediction of the escape rates by the thermal and non-thermal processes determines the long-term evolution of planetary atmospheres, it has long been a subject of interest in the aeronomy of planetary objects in our Solar system and more recently of interest in understanding the evolution of extrasolar planets. The particular feature that limits the modeling effort is that escape occurs in a rarefied region of an atmosphere, usually referred to as exosphere, where the gas flow is essentially non-equilibrium. In this region, continuum models break down so that molecular kinetic simulations are required to accurately predict the atmospheric flow structure and the escape rates. A detailed description of the kinetic Monte Carlo method used to study thermal (at high temperature) and nonthermal processes determining the rate of dissipation of planetary atmospheres will be presented. On all terrestrial planets, the dissipation timescales of primordial atmospheres are short. Although we cannot travel back in time to observe escape from early solar system terrestrial planets, observations and modeling of exoplanetary atmospheres under strong stellar radiation provide excellent opportunities to verify our theoretical understandings on this key process influencing planet evolution.